In Eqs. (7) and (8), e_x'', e_y'', and e_{xy}'' are the components of creep strain and the integration is over the thickness.

Consider a particular point in the plate. The stresses at this point vary continuously with time due to creep. The smooth stress time curve can be approximated by a series of finite steps, each of which consists of a constant stress period Δt followed by an instantaneous increment of stress. The increment of creep strains in the constant stress interval can be obtained as fully described in Ref. 1. The total creep strain at any time is the sum of creep strain increments of all the time steps. Hence, the creep strains and the equivalent forces readily can be calculated at the end of each time step. Our problem can then be solved in a manner similar to that used in Sec. 2.

At time $t = 0$ there is no creep strain, $\bar{q} = \bar{F} = 0$. An elastic solution is obtained first. At any subsequent time step, we use the previous deflection as the first trial deflection to start the iteration. In the iteration process, $\bar{q}$ and $\bar{F}$ remain unchanged while w,ϕ,q' and F approach their correct values.

4. Numerical Example

The procedure in Secs. 2 and 3 is applied to a 7075-T6 aluminum alloy square plate $\frac{1}{2}$ in. $\times$ 24 in. $\times$ 24 in. undergoing large creep deflection at 600°F. The lateral load q is uniform and equals 10 psi. The uniaxial creep characteristics of the material are approximated by $e_c = At^K \sinh(B\sigma)$, in which t is time in hours, and the material constants are $A = 5.25 \times 10^{-7}$, $B = 1.92 \times 10^{-3}$, $K = 0.66$, $E = 5.2 \times 10^6$, and $\nu = 0.32$. Two simply supported edge conditions are considered. One is with zero sectional forces, and the other with zero displacement. From the elastic solutions, the deflection of the plate due to a unit load at (ξ,η) is given by

$$w = \frac{a^2}{\pi^3 D} \sum_m \left(1 + m\pi \coth m\pi - \frac{m\pi y_1}{a}\coth\frac{m\pi y_1}{a} - \frac{m\pi\eta}{a}\coth\frac{m\pi\eta}{a}\right) \frac{\sinh(m\pi\eta/a)\sinh(m\pi y_1/a)\sin(m\pi\xi/a)\sin(m\pi x/a)}{m^3 \sinh m\pi} \quad (9)$$

where m denotes an integer, $y_1 = a - y$ and $y \geq \eta$. For $y < \eta$, y_1 is replaced by y and η by $a - \eta$. The plate is divided into 12 $\times$ 12 grid spacings. The influence coefficients for elastic moments are computed by finite differences giving, for instance,

$$Mx_{ij} = D[(1/\Delta x^2)(-w_{i-1,j} + 2w_{ij} - w_{i+1,j}) - (\nu/\Delta y^2)(-w_{i,j-1} + 2w_{ij} - w_{i,j+1})] \quad (10)$$

The stress influence coefficients for the plane stress problem are obtained by use of the computer program of Ref. 11 for both the stress free and immovable edges. The calculated results are shown in Figs. 1–3. The deflections for both cases are almost the same. The inplane forces vary differently.

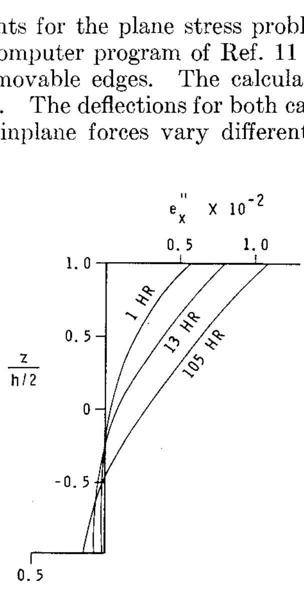

Fig. 3 Variation of creep strain at center of plate (stress free edge).

The numerical example exhibited an extremely rapid convergence. The first time increment was taken to be 0.0001 hr. The subsequent increment was doubled in each step. For a relative accuracy of 0.0002%, three cycles of iteration were enough for most steps and in no case exceeded five cycles. The total computing time took only 8 min, of which 5 min were spent in establishing the influence coefficients.

References

[1] Lin, T. H., *Theory of Inelastic Structures*, Wiley, New York, 1968, p. 65.

[2] Lin, T. H., "Bending of a Plate with Nonlinear Strain Hardening Creep," *Proceedings of the International Union of Theoretical and Applied Mechanics*, Springer Verlag, Berlin, 1962, pp. 215–228.

[3] Lackman, L. M., "Circular Plates Loaded into the Plastic Region," *Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers*, Vol. 90, EM 6, 1964, pp. 4155–4157.

[4] Lin, T. H. and Ganoung, J. K., "Bending of Rectangular Plates with Nonlinear Creep," *International Journal of Mechanical Sciences*, Vol. 6, 1964, pp. 337–348.

[5] Lin, T. H. and Ho, E., "Elasto-Plastic Bending of a Rectangular Plate," *Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers*, Vol. 94, EM 1, 1968, pp. 199–210.

[6] Levy, S., "Bending of Rectangular Plates with Large Deflection," Rept. 737, 1942, NACA.

[7] Chien, W. Z. and Yeh, K. Y., "On the Large Deflection of Rectangular Plate," *Proceedings of the Ninth International Congress of Applied Mechanics*, Brussels, Belgium, Vol. 6, 1957, pp. 403–412.

[8] Iygar, K. T. S. and Naqvi, M. M., "Large Deflections of Rectangular Plates," *Journal of Nonlinear Mechanics*, Vol. 1, 1966, pp. 109–122.

[9] Murray, D. W. and Wilson, E. L., "Finite Element Post Buckling Analysis of Thin Elastic Plates," *Proceedings of the Second Methods in Structural Mechanics Conference*, April 1969.

[10] Naghdi, P. M., "Bending of Elastoplastic Circular Plates with Large Deflection," *Transactions of the ASME, Ser. E, Journal of Applied Mechanics*, Vol. 16, 1952, pp. 1–7.

[11] Wilson, E. L., "Structural Analysis of Axisymmetric Solids," *AIAA Journal*, Vol. 3, No. 12, Dec. 1965, pp. 2269–2274.

Penetration of the Flame Front through a Fine Metal Layer in a Solid Propellant

S. S. Novikov* and Yu. S. Ryazantsev†
Soviet Academy of Sciences, Moscow, U.S.S.R.

In spite of the existence of a great number of works dealing with the investigations of nonsteady solid combustion, a comprehensive solid combustion theory is not available which takes into account the total amount of known experimental data on a combustion mechanism. The aforementioned experimental data comprise, for example, some results on the space extension of the heat release region, on the stability conditions of a combustion process, on the combustion of solid propellant under nonsteady pressure variations, and on the conditions of the extinction and ignition. The insufficiency of existing theories is caused mainly by the difficulties in experimental investigation of the combustion process.

Received December 16, 1969; revision received February 24, 1970.

* Senior Research Scientist, Institute of Chemical Physics.
† Senior Research Scientist, Institute of Problems in Mechanics.

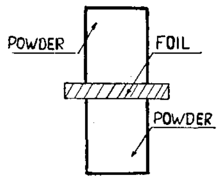

Fig. 1 Sample.

There are some methods of investigation of the nonsteady combustion described in the literature such as the method of pressure drop,[1] the method of limit diameter,[2] the method of combustion zone cooling by means of liquid coolant injection,[3] the method of T-burner,[4] and others.

In many cases a comparison between experimental and theoretical results is difficult because it is difficult to separate the effects directly connected with the combustion process from the accessory ones, such as the influence of the gas-dynamic field, the possibility of mechanical destruction of specimens, and the influence of acoustic properties of media and deviation from one-dimensional conditions.

The authors have previously suggested a method of investigation of the nonsteady combustion regime of powder.[5,7] The method enables us to investigate the extinction process in the absence of gasdynamic effects.

The present paper describes a new experimental method of investigation of nonsteady solid combustion, viz., the method of the combustion wave running through a fine metal plate in a solid propellant. The experiment is schematically shown in Fig. 1. The copper foil of 11 mm diam and 14μ thickness was embedded between two cylindrical nitroglycerine powder specimens of 10 mm diam and 10 mm height. Close thermal contact between powder and the metal foil was achieved by lubrication of the contact surface with acetone. To remove the acetone film, the sample was kept for several days in open air.

In a control series of experiments it was found that a small quantity of acetone had no influence on the results obtained. In the first series of the experiments the side surface of the specimens was covered by a polychlorvinyl varnish film.

The experiments were carried out in a constant pressure bomb with a nitrogen atmosphere. The upper specimen of powder was ignited at the top. After ignition, when the distance between combustion front and metal foil is greater than the characteristic thickness of the thermal layer in the solid propellant, the combustion front propagates steadily with velocity corresponding to a given pressure and initial temperature. When the combustion front approaches the metal foil the combustion regime becomes unsteady. Depending on pressure, initial temperature, and foil thickness, after the burning of the upper specimen the flame is extinguished or the heat flow penetrating through metal foil ignites the second specimen and combustion goes on.

If the pressure equals 1 atm and the specimen's initial temperature is within the interval 20–100°C the combustion wave does not run through copper foil of 14μ thickness and aluminum foil of 12μ thickness. The results were obtained mainly at room temperature. The results are given in Table 1. In the table the symbol N_1 marks a nitroglycerine powder whose combustion rate is greater than that of the powder N. In the experiments the foil thickness, kinds of metal and powder and coating of specimens were varied. For the analysis of the experimental results it is relevant to introduce the quantity $\mathcal{P}$, denoting the ratio of the number of experiments in

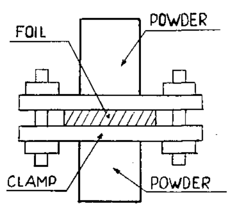

Fig. 2 Sample with clamp.

Table 1 Experimental effect of pressure on penetration of combustion wave through a metal foil

Number of series	Kinds of powder	Kind of metal	Foil thickness, μ	Foil diameter, mm	Pressure $\mathcal{P}$ atm	$\mathcal{P}(p)$
I[a]	$N \rightarrow N$	Cu	14	11	1.15	0
					20.25	0
					30	0.3
					40	1
					50	1
					100	1
II[a]	$N \rightarrow N$	Al	12	11	1.15	0
					20	0.5
					25	1
					30	1
III[a]	$N \rightarrow N_1$	Cu	14	11	25	0
					30	0.5
					40	1
	$N_1 \rightarrow N$	Cu	14	11	20	0
					25	0
					30	0.5
					40	1
	$N_1 \rightarrow N_1$	Cu	14	11	25	0
					30	0.3
					40	1
IV[a]	$N \rightarrow N$	Cu	50–200	11	100	0
V[b]	$N \rightarrow N$	Cu	14	11	20	0
					30	1
					40	1
					100	1
VI[c]	$N \rightarrow N$	Cu	14	11	1	0
					30	0
					40	0
					50	0
					60	0
					80	0.6
					120	1
VII[d]	$N \rightarrow N$	Cu	14	11	20	0.12
				15	25	0.3
					30	0.8
					40	0.9
					50	0.8
					80	1
					100	1

[a] Lacquer coating.
[b] Plexiglas clamp.
[c] Plexiglas coating.
[d] Without coating.

which the combustion wave ran through metal foil to the total number of experiments at given conditions.

A peculiar feature of the experiments should be noted. The small pressure intervals $p_* \pm \Delta p$ exists in all the series of the experiments (except the forth series). When the pressure is greater than $p_* + \Delta p$ practically all the experiments lead to the ignition of the lower specimen. When the pressure is smaller than $p_* - \Delta p$ the combustion wave does not run through the metal foil. The pressure value $p_* \sim$ 25–30 atm corresponds to the foil's 14 thickness. The value $\mathcal{P}(p_*)$ equals $\frac{1}{2}$ here.

Let us consider some features of the combustion wave running a metal foil. When the combustion front runs through a foil of 11 mm diam there is practically no residue

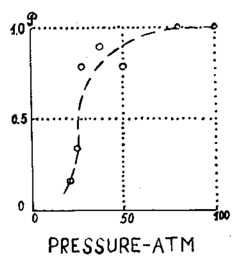

Fig. 3 Typical experimental results.

of the foil. In some experiments the diameter of foil was equal 15 mm and the possibility of flame propagation along the side of the specimen was excluded. In this case a round hole in the foil was observed after the combustion wave running through the metal foil. The hole diameter was equal to 11 mm. It was assumed that the foil was destroyed after the combustion wave passed through the foil. Special experiments were carried out with an ebonite or plexiglas specimen instead of lower powder specimen. In these cases an extinction took place and the foil was unharmed.

A special experiment was also made to investigate the effect of cohesion of the foil with the side coating of the specimens (series V). In this experiment the foil area was greater than the cross section of the specimen. The edge of the foil was squeezed by two special plexiglas plates which were fitted on propellant specimens and were fixed on both specimens. The clamp is illustrated schematically in Fig. 2. No essential influence of the clamp upon the value of critical pressure interval was observed. It is necessary to note that if the combustion front does not pass through a metal foil, a fine slab of the unburned powder remains at the metal. (At $p =$ 20 atm the foil thickness is less than 0.03 mm.) It means that the extinction takes place before the combustion front reaches the upper surface of the metal foil.

When the combustion wave runs through a metal foil a delay of ignition is observed. The lower specimen ignition does not occur simultaneously with arrival of the luminous combustion front at the foil. If the pressure increases the ignition delay time decreases.

The experimental results in the case of series VII are shown in Fig. 3. In this series of experiments both powder specimens had no side coating. In Fig. 3 the function $\mathcal{P}(p)$ has a specific form.

It is necessary to note that when a foil thickness is given, "the probability of combustion wave running through a foil" increases with pressure and the combustion rate also increases, as is known. Therefore the characteristic thermal layer in a specimen decreases.

It is possible to give a qualitative explanation of the phenomenon of combustion wave running through a foil. The thermal diffusivity of foil is higher than that of the powder and we can assume the foil temperature does not depend on the spatial coordinate. In this approximation the balance of thermal energy of the foil is as follows:

$$\lambda(\zeta_+ - \zeta_-) = \rho c l(\partial T/\partial t)$$

where ρ is foil density, c is foil specific heat, λ is thermal conductivity of powder, l is foil thickness, T is the mean foil temperature, ζ_+, ζ_- are gradients of temperature at the upper and lower surfaces of the foil in the specimens.

There is a jump of the temperature gradient at the foil. Therefore at the moment when the combustion front reaches the upper foil surface the temperature gradient at the lower foil surface is smaller than at the upper one. According to the theory, for example,[6] the propagation of combustion wave in solid propellant (powder N) is possible only if the temperature gradient at the burning surface exceeds a certain critical minimum value ζ_m. If the temperature gradient at the lower foil surface is smaller than the critical value the ignition of the lower specimen is impossible. If the temperature gradient at the lower foil surface is greater than the critical value the combustion wave penetrates through the foil. Since the range of allowable temperature gradients at the burning surface increases with increasing pressure, the ζ_- value remains less than ζ_m at $p < p_*$ and the ignition of the lower specimen is thus impossible. If $p > p_*$ then $\zeta_- > \zeta_m(p)$ and the combustion wave penetrates through the metal foil.

References

[1] Ciepluch, C. C., "Effects of Rapid Pressure Decay on Solid Propellant Combustion," *ARS Journal*, Vol. 31, No. 11, Nov. 1961, pp. 1584–1586.

[2] Andreev, K. K., *The Thermal Decomposition and Combustion of Explosives*," Izdatel'stvo "Nauka," Moscow, 1966, pp. 109–123 (in Russian).

[3] Slocum, R. W., "Results of Solid Rocket Motor Extinguishing Experiments," *AIAA Journal*, Vol. 1, No. 6, June 1963, pp. 1419–1420.

[4] Horton, M. D., "Use of the One-Dimensional T-Burner to Study Oscillatory Combustion," *AIAA Journal*, Vol. 2, No. 6, June 1964, pp. 1112–1118.

[5] Novikov, S. S. et al., "On the Investigation of a Powder Extinction by the Method of "Freezing" the Combustion Zone," *DAN SSSR*, Vol. 180, No. 6, 1968, pp. 1391–1394 (in Russian).

[6] Novikov, S. S. and Ryazantsev, Yu. S., "On Analysis of Powder Extinction Conditions," *Journal PMTF*, No. 2, 1969 (in Russian).

[7] Novikov, S. S. and Ryazantsev, Yu. S., "Extinction of Propellant near the Contact with a Metal," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 358–359.

Subcritical and Supercritical Boundary Layers

JOHN L. STOLLERY* AND WILBUR L. HANKEY†
Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio

MOMENTUM integral techniques have been used for many years to solve the laminar and turbulent boundary-layer equations and have met with a considerable measure of success. In attempting to apply such methods to an ever-increasing range of Mach numbers and wall temperature ratios, some difficulty has arisen because under certain supersonic or hypersonic cold wall conditions the boundary layer tends to thin in an adverse pressure gradient and it is difficult to match this behavior to the development of the compression process in the external stream. Crocco[1] was the first to recognize this dilemma and he classified boundary layers as either subcritical or supercritical, depending on whether the displacement thickness increased or decreased in an adverse (i.e., positive) pressure gradient. This classification was adopted in the Crocco-Lees[2] paper and more recently in the Lees-Reeves[3] method of solving the laminar boundary-layer equations. Lees and Reeves found it necessary to introduce a jump from supercritical to subcritical behavior in order to reach separation with hypersonic cold wall flows.

A great deal of discussion has centered on the physical causes of these difficulties and much ingenuity has been used in pushing the supercritical boundary just out of sight. For example, it has been shown[2] that the choice of boundary-layer thickness (δ) has a considerable influence on the position of the critical line; the larger the value of δ the more hypersonic stream tubes are included and the more supercritical the boundary layer becomes. The neglect of the normal pressure gradient (certainly an important feature of the real flow) also has been cited as the main cause of difficulty, and Holden[4] has shown how the introduction of centrifugal effects effectively removes the supercritical boundary from regions of interest. The main aim of this Note is to re-examine a few of the various momentum integral methods and demonstrate that the critical boundary depends entirely on the formulation

Received January 21, 1970.

* Visiting Research Associate, Imperial College, London, England. Member AIAA.

† Aerospace Engineer, Hypersonic Research Laboratory. Member AIAA.